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Derivation of a tube solar collector transmittance for beam and diffuse radiation

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Abstract

An expression of the total directional ray transmittance for a tube solar collector cover glass has been derived. Numerical hemispheric integration has been made for both beam and diffuse radiation. From the equivalent solar altitude angle we derived an expression that gives the equivalent altitude which can be used to estimate the average transmittance of a tubular cover glass using the mean beam transmittance expression.

Keywords: Mean solar transmittance; Beam solar radiation; Diffuse solar radiation; Equivalent angle; Numerical derivation

1. Introduction

The transmittance of a flat plate solar collector cover glass, for the beam solar radiation, has been derived early by Dietz [1,2]. Later and always for the flat plate case, Brandemuehl and Beckman [3] calculated both ground and sky diffuse solar radiation transmittance. To avoid the hemispherical integration, the authors proposed two formulas giving the "effective incident angle" which can be used of the diffuse transmittance estimation, applying the beam algorithm.

At the same period, Felske [5] published a method to determine the average transmittance of a tube solar collector cover glass for a beam solar radiation. Neglecting the solar ray refraction effect, Felske derivation was limited to the case where the w/D ratio is equal to unity. The mean global transmittance is then calculated after integration on the surface tube. For the longitudinal direction the Felske hypothesis are the same as those taken for the flat plate transmittance derivation [3,5]. Further works extend the analysis introducing the ray refraction effect [6,7], but maintaining the w/D ratio equal to unity.

In this paper, the average transmittance of a tube solar collector cylindrical cover glass to the beam radiation, for different collector tilt angles and w/D ratios, has been evaluated using Felske [5] approach but considering the solar ray refraction effects. Neglecting the thermal dependence of the physical properties of the materials and assuming a tube length as infinite the average transmittance equation, for the beam and diffuse radiation, has been integrated numerically for both sky and ground sources.

On the other hand, since the use of the equivalent incidence angle is not adapted for a curved surfaces, the equivalent solar altitude has been chosen as parameter of similarity.

2. Optical properties of a cylindrical cover glass

Let us consider a tube solar collector with a flat plate absorber. Applying the relations derived by Fresnel, the reflectance of the first interface of a tubular cover glass (Fig. 1) is given by [4,7]:

$$r_1 = (r_{1//} + r_{1\perp})/2 \tag{1}$$

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Nomenclature

A	cover glass area	m^2
D	external cover diameter	m
E	collected solar energy	W
I	solar radiation	$W \cdot m^{-2}$
L	collector length	m
R	external cover radius	m
e	cover thickness	m
h	solar altitude, degrees	
k	extinction coefficient	m^{-1}
l	optical distance	m
n	average refractive index	
r	$= w/D$ ratio	
r_1 and r_2	interface reflectivity	
w	collector width	m

Greek letters

Δ	$= \ddot{\tau}_{d1} - \ddot{\tau}_{d2}$	
α	solar azimuth, global absorptivity	degrees
β	collector tilt angle	degrees
γ	collector azimuth	degrees
θ	solar incidence angle	degrees
ρ	global cover reflectance	
τ	global cover transmittance	
$\ddot{\tau}_{d1}$	cover glass average transmittance using the hemispherical integration	
$\ddot{\tau}_{d2}$	cover glass average transmittance using the equivalent solar altitude	
ξ, η	projections of the incidence angle	degrees
ψ	integration angle	degrees

Subscripts

b and d	for beam and diffuse of the solar radiation, respectively
// and $\perp$	for parallel and perpendicular radiation component, respectively

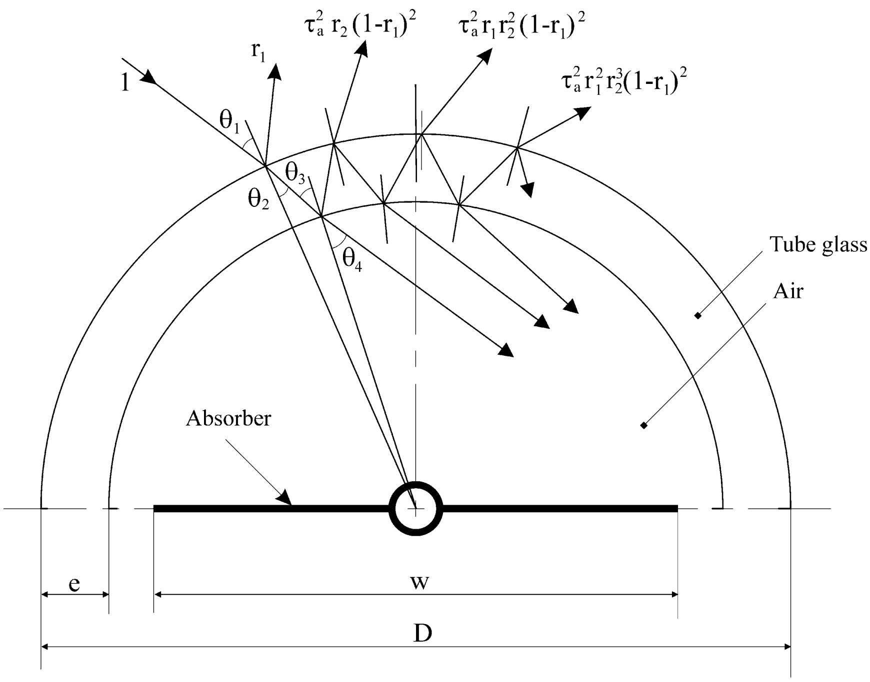

Fig. 1. Collector cross section and ray tracing application.

with:

$$r_{1//} = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \tag{2}$$

$$r_{1\perp} = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \tag{3}$$

and

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{(Snell's law)} \tag{4}$$

For the second interface, we have:

$$r_2 = (r_{2//} + r_{2\perp})/2 \tag{5}$$

with:

$$r_{2//} = \frac{\tan^2(\theta_3 - \theta_4)}{\tan^2(\theta_3 + \theta_4)} \tag{6}$$

$$r_{2\perp} = \frac{\sin^2(\theta_3 - \theta_4)}{\sin^2(\theta_3 + \theta_4)} \tag{7}$$

$$n_2 \sin\theta_3 = n_1 \sin\theta_4 \tag{8}$$

On the other hand, we can write (see Fig. 1):

$$D\sin\theta_2 = (D-2e)\sin\theta_3 \tag{9a}$$

from which:

$$D\sin\theta_1 = (D-2e)\sin\theta_4 \tag{9b}$$

Using the Ray Tracing Method, the optical properties of a cover, for the parallel component of the radiation, can be expressed by [4–6]:

$$\begin{aligned}
\tau_{//} &= \tau_a(1-r_{1//})(1-r_{2//})\sum_{i=1}^{\infty}\left(\tau_a^2 r_{1//} r_{2//}\right)^{i-1} \\
&= \tau_a\frac{(1-r_{1//})(1-r_{2//})}{1-\tau_a^2 r_{1//} r_{2//}} \\
\rho_{//} &= r_{1//} + \tau_a^2 r_{2//}(1-r_{1//})^2\sum_{i=1}^{\infty}\left(\tau_a^2 r_{1//} r_{2//}\right)^{i-1} \\
&= r_{1//} + \frac{\tau_a^2 r_{2//}(1-r_{1//})^2}{1-\tau_a^2 r_{1//} r_{2//}}
\end{aligned} \tag{10}$$

Assuming that the cover is a partially transparent medium, the application of the Lambert–Bouguer's law [4] gives (see Fig. 2):

$$\tau_a = \mathrm{e}^{-kd/\cos\alpha_2} \tag{11a}$$

where:

$$d = \frac{D}{2}\left[\cos\theta_2 - (1-2e/D)\cos\theta_3\right] \tag{11b}$$

The cover properties for the perpendicular component are described by similar formulas. The transmittance, the absorptance and the reflectance of the cover are obtained from the average of the respective properties for both components.

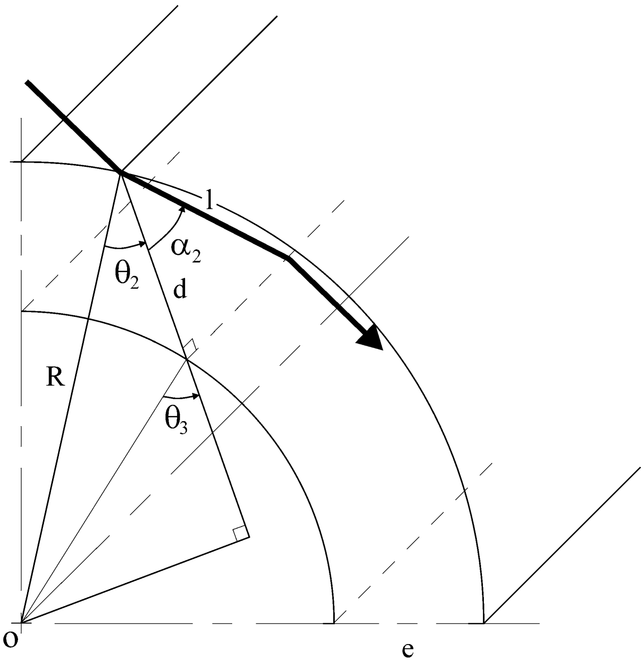

Fig. 2. Optical distance for a tubular cover glass.

3. Incidence angle

Let us now consider a tube solar collector exposed to solar radiation with a tilt angle β, from the horizontal, and an azimuth angle γ, from the south direction (Fig. 3). If $\vec{s}$ represents the unit vector of the solar ray direction and $\vec{n}$ the unit vector normal to the cover, at the point of the incidence solar ray, the incidence angle θ will be given by:

$$\begin{aligned}
\cos\theta &= \vec{n}\cdot\vec{s} \\
&= \cos h\sin(\alpha+\gamma)\sin\psi \\
&\quad + \left[\sin\beta\cos h\cos(\alpha+\gamma) + \cos\beta\sin h\right]\cos\psi
\end{aligned} \tag{12}$$

or

$$\cos\theta = X\sin\psi + Y\cos\psi \tag{13}$$

i'', j'' and k'' being the unit vector of the Cartesian axis system $X''OY''$, the projections of the incidence angle on the planes $Y''OZ''$ and $X''OZ''$, namely ξ and η are, respectively, given by:

$$\tan\xi = \frac{\vec{j''}\cdot\vec{s}}{\vec{k''}\cdot\vec{s}} = \frac{\cos h\sin(\alpha+\gamma)}{\cos h\sin\beta\cos(\alpha+\gamma)+\sin h\cos\beta} \tag{14}$$

$$\tan\eta = \frac{\vec{i''}\cdot\vec{s}}{\vec{k''}\cdot\vec{s}} = \frac{-\cos h\cos\beta\cos(\alpha+\gamma)+\sin h\sin\beta}{\cos h\sin\beta\cos(\alpha+\gamma)+\sin h\cos\beta} \tag{15}$$

4. Average transmittance to beam radiation

The incident energy transmitted by the cover and received by an elementary surface $\mathrm{d}A$ of the collector is given by:

$$\mathrm{d}E = I_b\tau(\theta)\vec{s}\cdot\vec{n}\,\mathrm{d}A = LRI_b\tau(\theta)\cos\theta\,\mathrm{d}\psi \tag{16}$$

and the energy which would be received by the same elementary surface in absence of the cover, can be expressed by:

$$\mathrm{d}E_t = I_b\vec{s}\cdot\vec{n}\,\mathrm{d}A_t = LI_b\cos\theta_t\,\mathrm{d}w \tag{17}$$

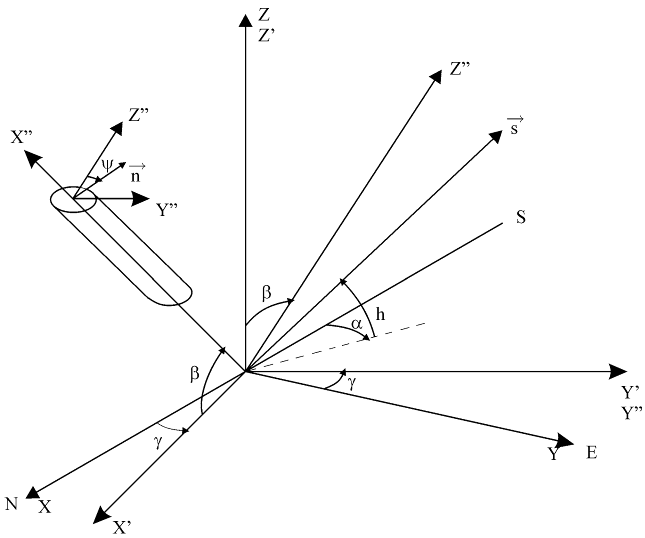

Fig. 3. Schematic representation of a tube collector exposed to the solar radiation.

Then, the average transmittance of the cover will be equal to:

$$\bar{\tau}(\theta) = \frac{1}{E_t}\int_{\psi_1}^{\psi_2} \mathrm{d}E = \frac{1}{Lw I_b \cos\theta_a}\int_{\psi_1}^{\psi_2} LRI_b \tau(\theta)\cos\theta\,\mathrm{d}\psi \tag{18}$$

After rearranging, we obtain:

$$\bar{\tau}(\theta) = \frac{R}{wY}\int_{\psi_1}^{\psi_2} \tau(\theta)(X\sin\psi + Y\cos\psi)\,\mathrm{d}\psi \tag{19}$$

The integration limits Ψ_1 and Ψ_2 being equals to:

$$\begin{cases}\psi_1 = \xi \\ \psi_2 = \xi - \theta_1\end{cases} \tag{20}$$

The incidence angle θ_1 is the solution of the following system of equations:

$$\begin{cases}\dfrac{\sin\theta_1}{\sin\theta_4} = 1 - 2e/D \\ \dfrac{\sin\theta_4}{w/2} = \dfrac{1}{R}\cos\Big[\theta_1 - \xi - \theta_4 + \Big(\arcsin\Big(\dfrac{n_1}{n_2}\sin\theta_4\Big) \\ \qquad - \arcsin\Big(\dfrac{n_1}{n_2}\sin\theta_1\Big)\Big)\Big]\end{cases} \tag{21}$$

This system of equations is solved numerically using an iterative method [9].

Fig. 4 represents the average transmittance of a tubular glass cover exposed to the beam radiation as function of the collector tilt angle β, for $\alpha = 0$ and $h = 0$ to 90 degrees. We can see the average transmittance vary asymptotically to its maximum, with the tilt angle, which is 0.9 for the considered tube glass.

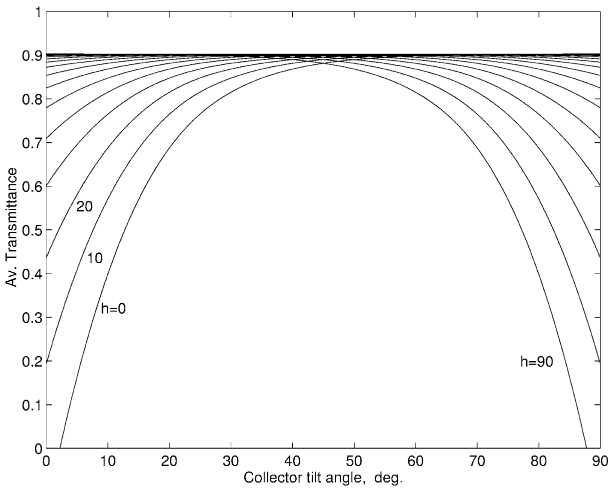

Fig. 4. Average Transmittance of a tube cover glass for the beam solar radiation at $a = 0$ (with $D = 0.077$, $w = 0.038$ and $e = 0.0025$).

5. Average transmittance to diffuse radiation

For the diffuse radiation case, Eq. (19) becomes:

$$\bar{\tau}(\theta) = \frac{\int_{\cap}\int_{\psi_1}^{\psi_2}\mathrm{d}E}{\int_{\cap}\mathrm{d}E_t} = \frac{\int_{\cap}\int_{\psi_1}^{\psi_2} LDI_d\tau(\theta)\cos\theta\,\mathrm{d}\Omega\,\mathrm{d}\psi}{\int_{\cap} LI_d\cos\theta_a\,\mathrm{d}\Omega\,\mathrm{d}w} \tag{22}$$

Replacing the solid angle Ω by its expression, we get:

$$\bar{\tau}(\theta) = \frac{\int_{\Delta\alpha}\int_{\Delta h}\int_{\psi_1}^{\psi_2} LDI_d\tau(\theta)\cos\theta\cos h\,\mathrm{d}\alpha\,\mathrm{d}h\,\mathrm{d}\psi}{\int_{\Delta\alpha}\int_{\Delta h} LI_d\cos\theta_a\cos h\,\mathrm{d}\alpha\,\mathrm{d}h\,\mathrm{d}w} \tag{23}$$

If we assume that the diffuse radiation is isotropic, the average transmittance of the cylindrical cover for the sky diffuse radiation is given by:

$$\begin{aligned}\bar{\tau}(\theta) = {} & \frac{D}{w}\int_{\alpha=0}^{2\pi}\int_{h=hm}^{\pi/2}\int_{\psi_1}^{\psi_2}\tau(\xi,\eta) \\ & \times \cos^2 h\sin(\alpha+\gamma)\sin\psi\,\mathrm{d}\alpha\,\mathrm{d}h\,\mathrm{d}\psi \\ & \times \Big[\int_{\alpha=0}^{2\pi}\int_{h=hm}^{\pi/2}\cos h\big[\sin\beta\cos h\cos(\alpha+\gamma) \\ & \qquad + \cos\beta\sin h\big]\,\mathrm{d}\alpha\,\mathrm{d}h\Big]^{-1} \\ & + \frac{D}{w}\int_{\alpha=0}^{2\pi}\int_{h=hm}^{\pi/2}\int_{\psi_1}^{\psi_2}\tau(\xi,\eta)\big[\sin\beta\cos h\cos(\alpha+\gamma) \\ & \qquad + \cos\beta\sin h\big] \\ & \times \cos h\cos\psi\,\mathrm{d}\alpha\,\mathrm{d}h\,\mathrm{d}\psi \\ & \times \Big[\int_{\alpha=0}^{2\pi}\int_{h=hm}^{\pi/2}\cos h\big[\sin\beta\cos h\cos(\alpha+\gamma) \\ & \qquad + \cos\beta\sin h\big]\,\mathrm{d}\alpha\,\mathrm{d}h\Big]^{-1}\end{aligned} \tag{24}$$

$h_m = 0$ for $\alpha = -\pi/2$ to $\pi/2$ (front face)

$h_m = \beta$ for $\alpha = \pi/2$ to $3\pi/2$ (back face)

The average transmittance of the ground diffuse radiation is estimated by replacing the integration limits of the solar altitude, in Eq. (24). So that, h varies from $-\beta$ to 0 and α varies from $-\pi/2$ to $\pi/2$.

In Fig. 5, we represent the average transmittance curves for both sky and ground diffuse radiation versus the tilt angle β, for different w/D ratios. We can notice that the ground radiation transmittance varies asymptotically when the sky radiation transmittance is quasi constant and present a maximum around 45 degrees of tilt angle; which is conform to the view factor variation.

Comparing the results obtained for the diffuse and beam radiation at $\alpha = 0$, we carry out the matrixes of equivalent solar beam altitude for both sky and ground diffuse radiation. Processing these data, in the least square sense [8,9], we obtained that the equivalent solar altitude could be determined, with a good accuracy, using the following relations, namely:

$$h_e = 136.4 - \left[0.063(r - 0.5) + 1.3164\right]\beta + 0.0031\beta^2 + 0.46238\left[1 - e^{9.04259(r-0.5)}\right] \quad (25)$$

for the sky diffuse radiation and

$$h_e = 158.7 - \left\{1.5616 - 0.067051\,\mathrm{Log}\left[1 + 110.6895(0.9351 - r)\right]\right\}\beta \quad (26)$$

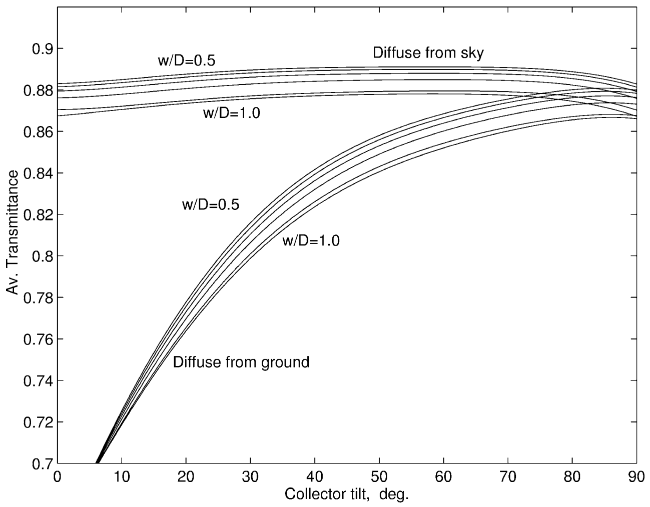

Fig. 5. Average Transmittance of a tube cover glass for the diffuse solar radiation for various w/D ratios.

for the ground diffuse radiation. The collector tilt angle β is expressed in degrees.

In Tables 1 and 2 are reported the average transmittance of a tube cover glass for both sky and ground diffuse solar radiation, using the equivalent altitude equations and the hemispherical integration methods, for different w/D ratios and collector tilt angles. The results shows that, for a tilt angle greater than 20 degrees, the error found when using the simplified method is always lower than 0.1% for the sky diffuse and 0.4% for the ground diffuse radiation.

6. Conclusion

The solar Transmittance expression of a cylindrical cover glass, for both sky and ground diffuse radiation, has been derived. The average solar transmittance is obtained by numerical integration using nonlinear regression analysis. The equivalent solar beam altitude has been deduced in the two cases and mathematical expressions have been derived. When calculating a global solar transmittance of a tube cover is necessary to evaluate the transmittance of all solar radiation components. Applying the present results, the global tube cover transmittance, for a given altitude and azimuth, can be estimated using only beam algorithm. The equivalent solar altitude being evaluated by Eqs. (25) and (26). As discussed above, the estimation error is less then 0.4%, which can be considered as a good accuracy for solar applications (for example, the pyranometer accuracy is generally up than 2%, [4]).

Table 1
Averance Transmittance estimation, of a Tube Cover Glass ($n = 1.52$ and $k = 16\ \mathrm{m}^{-1}$), for sky Diffuse Radiation by the hemispheric integration ($\ddot{\tau}\,\tau_{d1}$) and the direct derivation ($\ddot{\tau}\,\tau_{d2}$) methods

β	$r = 0.5$			$r = 0.7$			$r = 0.9$		
	$\ddot{\tau}\,\tau_{d1}$	$\ddot{\tau}\,\tau_{d2}$	Δ (%)	$\ddot{\tau}\,\tau_{d1}$	$\ddot{\tau}\,\tau_{d2}$	Δ (%)	$\ddot{\tau}\,\tau_{d1}$	$\ddot{\tau}\,\tau_{d2}$	Δ (%)
10	0.8850	0.8844	0.06	0.8813	0.8802	0.11	0.8721	0.8746	−0.25
20	0.8876	0.8881	−0.05	0.8840	0.8840	0.00	0.8748	0.8764	−0.16
30	0.8895	0.8903	−0.07	0.8861	0.8863	−0.02	0.8771	0.8776	−0.05
40	0.8906	0.8915	−0.09	0.8873	0.8876	−0.03	0.8785	0.8782	0.02
50	0.8910	0.8919	−0.09	0.8878	0.8882	−0.03	0.8792	0.8786	0.06
60	0.8911	0.8916	−0.06	0.8880	0.8880	−0.01	0.8795	0.8786	0.09
70	0.8907	0.8906	0.01	0.8875	0.8872	0.03	0.8791	0.8783	0.08
80	0.8888	0.8886	0.01	0.8854	0.8855	−0.01	0.8769	0.8776	−0.07

Table 2
Averance Transmittance estimation of a Tube Cover Glass ($n = 1.52$ and $k = 16\ \mathrm{m}^{-1}$) for graound diffuse radiation by the hemispheric integration ($\ddot{\tau}\,\tau_{d1}$) and the direct derivation ($\ddot{\tau}\,\tau_{d2}$) methods

β	$r = 0.5$			$r = 0.7$			$r = 0.9$		
	$\ddot{\tau}\,\tau_{d1}$	$\ddot{\tau}\,\tau_{d2}$	Δ (%)	$\ddot{\tau}\,\tau_{d1}$	$\ddot{\tau}\,\tau_{d2}$	Δ (%)	$\ddot{\tau}\,\tau_{d1}$	$\ddot{\tau}\,\tau_{d2}$	Δ (%)
10	0.7253	0.7560	−3.07	0.7229	0.7410	−1.81	0.7194	0.7133	0.60
20	0.7777	0.7909	−1.30	0.7730	0.7805	−0.75	0.7655	0.7623	0.31
30	0.8161	0.8177	−0.16	0.8106	0.8104	0.02	0.8011	0.7973	0.37
40	0.8415	0.8383	0.33	0.8361	0.8330	0.32	0.8260	0.8223	0.37
50	0.8580	0.8540	0.40	0.8530	0.8499	0.30	0.8430	0.8402	0.27
60	0.8686	0.8661	0.25	0.8641	0.8627	0.14	0.8544	0.8531	0.13
70	0.8755	0.8753	0.03	0.8714	0.8723	−0.09	0.8620	0.8623	−0.03
80	0.8800	0.8822	−0.22	0.8762	0.8795	−0.33	0.8671	0.8689	−0.18

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